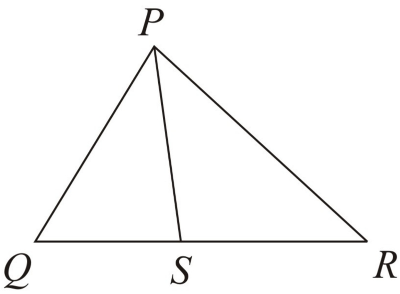
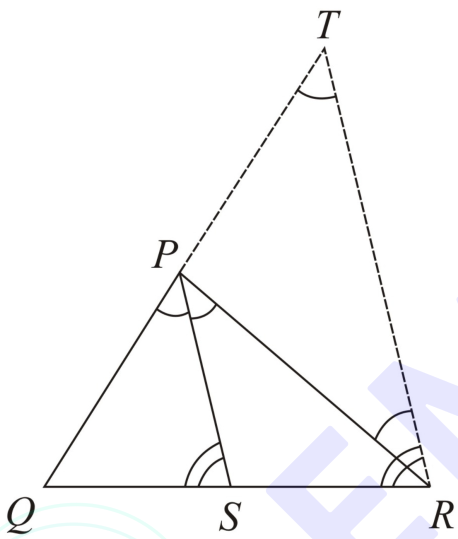
Exercise 6.6

Q.1. In the given figure, PS is the bisector of ∠QPR of ΔPQR. Prove that QSSR=PQPR.



Solution:



Given that, PS is angle bisector of ∠QPR.

Construct a line RT parallel to SP which meets QP produced at T. ∠QPS=∠SPR ......(1) ∠SPR=∠PRT (As PS||TR, alternate interior angles) ....(2) ∠QPS=∠QTR (As PS||TR, corresponding angles) .....(3) Using these equations, we may find ∠PRT=∠QTR from (2) and (3) So, PT=PR (Since ΔPTR is isosceles triangle)

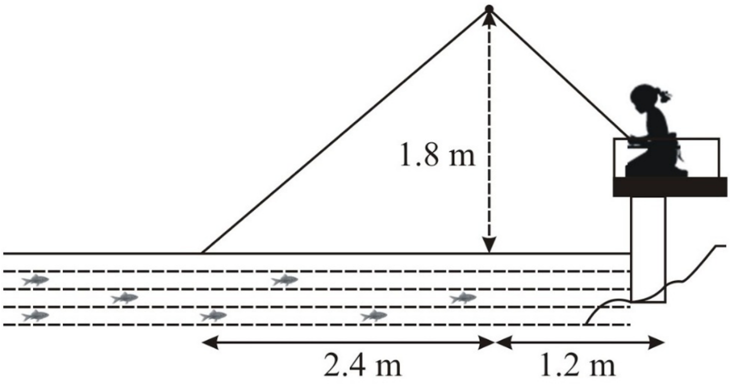
Now in ΔQPS and ΔQTR, ∠QSP=∠QRT (As PS||TR)

∠QPS=∠QTR (As PS || TR)

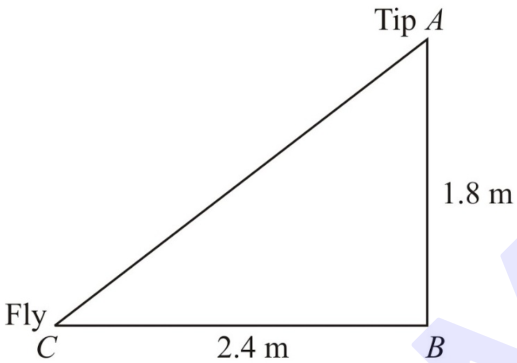
∠Q is common. ΔQPS~ΔQTR (by AAA property) So, QRQS=QTQP ⇒QRQS-1=QTQP-1 ⇒SRQS=PTQP ⇒QSSR=QPPT ⇒QSSR=PQPR

EMBIBE

Q.2. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see the given figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after12 seconds?



Solution:

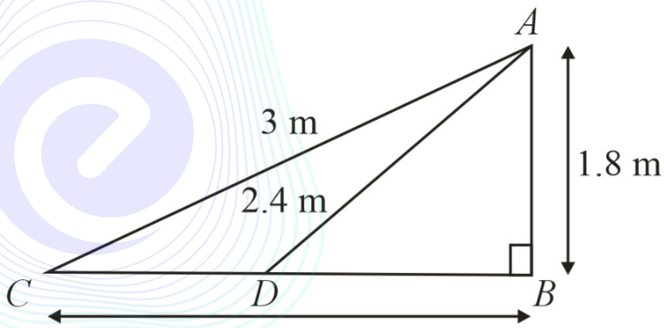


Let AB be the height of tip of fishing rod from water surface and BC be the horizontal distance of fly from the tip of fishing rod.

Then, AC is the length of string. AC can be found by applying Pythagoras theorem in ΔABC. AC2=AB2+BC2 AC2=1.82+2.42 AC2=3.24+5.76 AC2=9.00 Thus, length of string out is 3 m.

Now, she pulls string at rate of 5 cm per second.

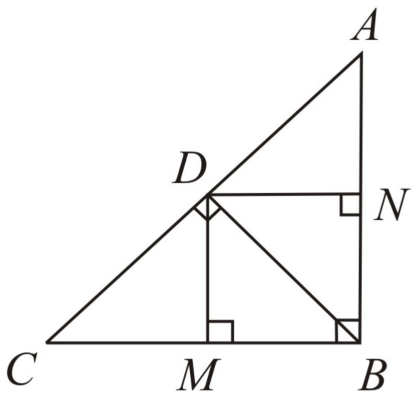
So, string pulled in 12 second=12×5=60 cm=0.6 m.



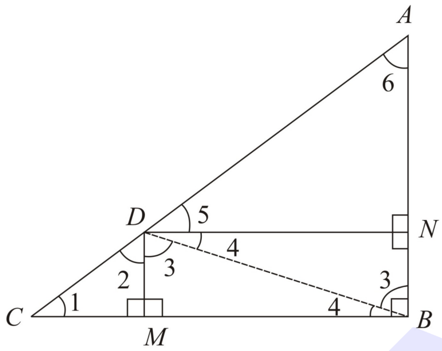
After 12 seconds, let us assume the fly to be at point D.

Length of string out after 12 second is AD. AD=AC- string pulled by Nazima in 12 second =3.00-0.6 =2.4 m In ΔADB, AB2+BD2=AD2 ⇒1.82+BD2=2.42 ⇒BD2=5.76-3.24=2.52 ⇒BD=1.587 m Horizontal distance of fly =BD+1.2 =1.587+1.2 =2.787 =2.79 m

Q.3. In the figure, D is a point on hypotenuse AC of ΔABC, such that BD⊥AC, DM⊥BC and DN⊥AB. Prove that : DM2=DN.MC



Solution: Let us join DB.



DN||CB, DM||AB

Therefore, DNBM is a parallelogram.

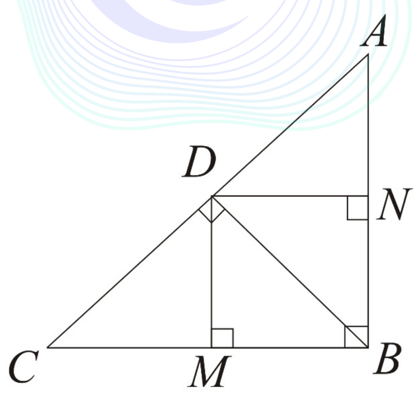
Since, ∠B is 90°, therefore, DNBM is a rectangle. Hence, DN=MB, DM=NB and ∠CDB=∠ADB=90° ∠2+∠3=90o…(1) In ΔCDM ∠1+∠2+∠DMC=180° ∠1+∠2=90°…(2) In ΔDMB ∠3+∠DMB+∠4=180° ∠3+∠4=90°…(3)

From equation (1) and (2)

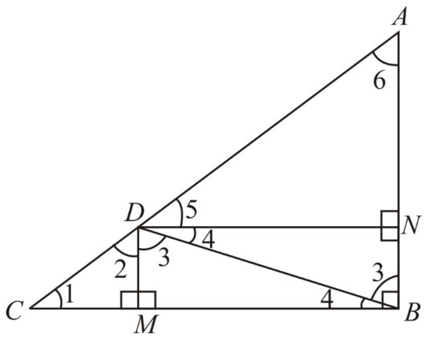
∠1=∠3

From equation (1) and (3) ∠2=∠4 So, ΔBDM~ΔDCM BMDM=DMMC ⇒DNDM=DMMC ⇒DM2=DN.MC Hence, proved.

Q.4. In the figure, D is a point on hypotenuse AC of ΔABC, such that BD⊥AC,DM⊥BC and DN⊥AB. Prove that: DN2=DM·AN



Solution: Let us join DB.



$DN \parallel CB$, $DM \parallel AB$

Therefore, DNBM is a parallelogram.

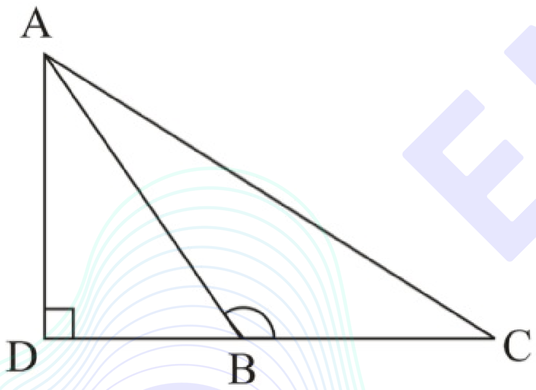
Since, $\angle B$ is $90°$. Therefore, DNBM is a rectangle. So, $DN=MB$, $DM=NB$ and $\angle CDB=\angle ADB=90°$ $\angle 4+\angle 5=90o$…(1)
In Δ ADN $\angle 5+\angle 6=90°$…(2)

From equation (1) and (2)

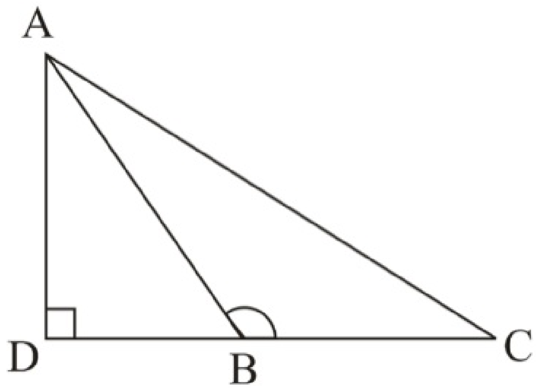
$\angle 4=\angle 6$

and $\angle DNA=\angle DNB=90°$ So, $\Delta ADN \sim \Delta BDN$ DNBN=ANDN $\Rightarrow$DNDM=ANDN (As $BN=DM$) $\Rightarrow$DN2=DM×AN Hence, proved.

Q.5. ABC is a triangle in which $\angle ABC>90o$ and $AD\perp CB$ produced. Prove that
AC2=AB2+BC2+2BC·BD.



Solution:



In ΔADB, applying Pythagoras theorem
AB2=AD2+DB2…(1)

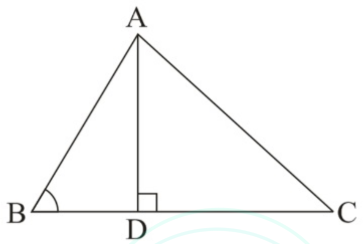
In ΔACD, applying Pythagoras theorem
AC2=AD2+DC2

⇒AC2=AD2+DB+BC2

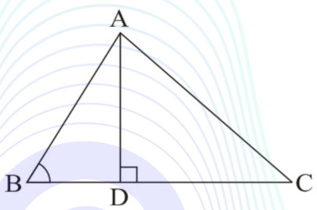
⇒AC2=AD2+DB2+BC2+2DB×BC

Now using equation (1)
AC2=AB2+BC2+2BC.BD

Q.6. In the figure given below ABC is a triangle in which ∠ABC<90o and AD⊥BC. Prove that AC2=AB2+BC2-2BC.BD.



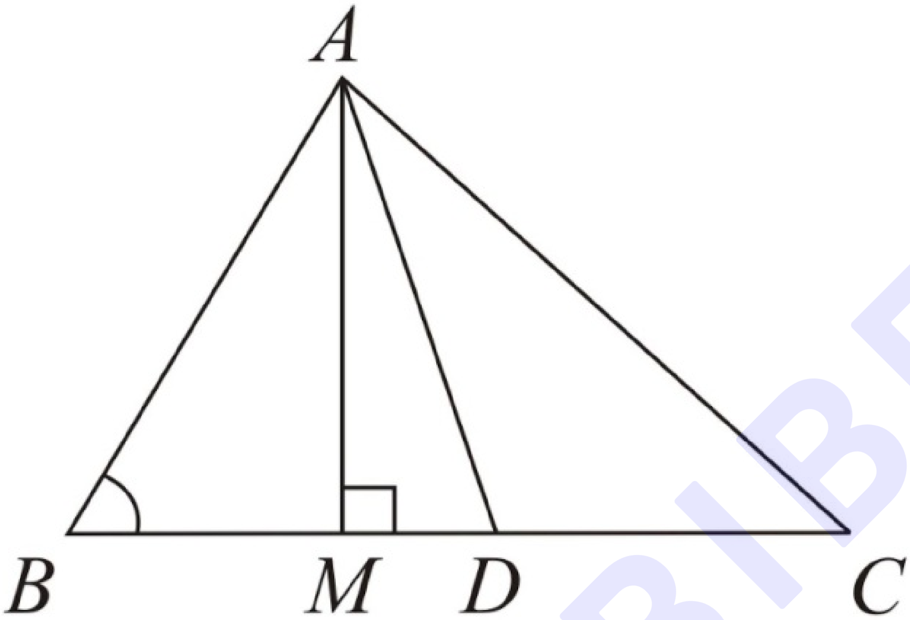
Solution:



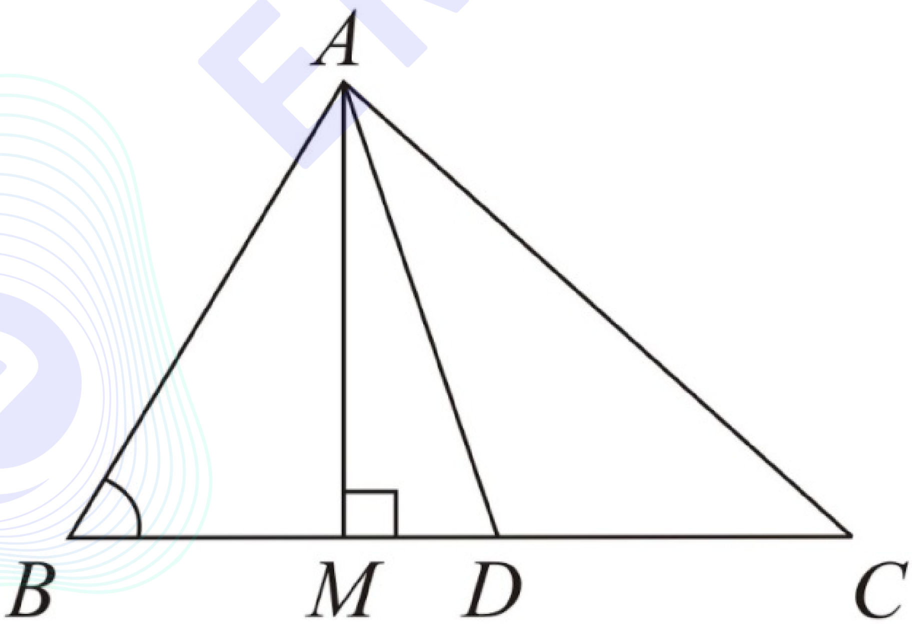
In △ADB, applying Pythagoras theorem we get:
AD2+DB2=AB2
⇒AD2=AB2-DB2…(1)
In △ADC, applying Pythagoras theorem we get:
AD2+DC2=AC2 ....(2)

Now using equation (1), we get:
AB2-BD2+DC2=AC2
⇒AB2-BD2+BC-BD2=AC2
⇒AC2=AB2-BD2+BC2+BD2-2BC.BD
Hence, AC2=AB2+BC2-2BC.BD.

Q.7. In the figure, AD is a median of a triangle ABC and AM⊥BC. Prove that :
AC2=AD2+BC·DM+BC22



Solution:



In ΔAMD, by using Pythagoras theorem,

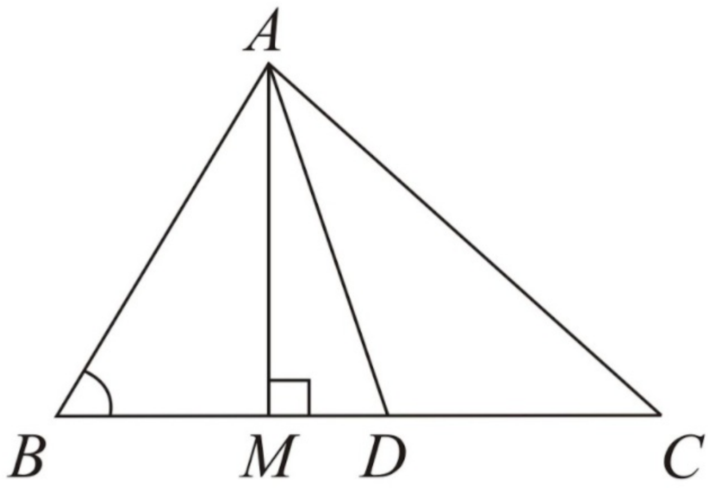
AM2+MD2=AD2…(1) In ΔAMC AM2+MC2=AC2…(2) ⇒AM2+MD+DC2=AC2
⇒AM2+MD2+DC2+2MD.DC=AC2 Using equation (1) we get, AD2+DC2+2MD.DC=AC2

Now using the result, DC=BC2

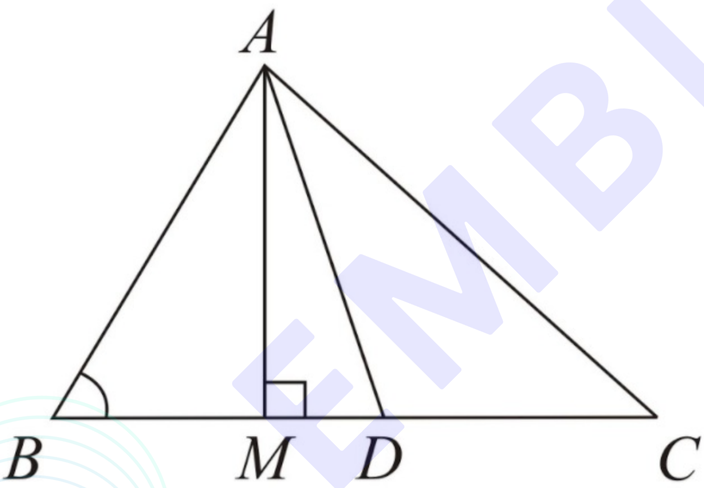
AD2+BC22+2MD.BC2=AC2

⇒AD2+BC22+MD×BC=AC2 Hence, AC2=AD2+BC·DM+BC22

Q.8. In the given figure, AD is a median of a triangle ABC and AM⊥BC. Prove that AB2=AD2-BC·DM+BC22



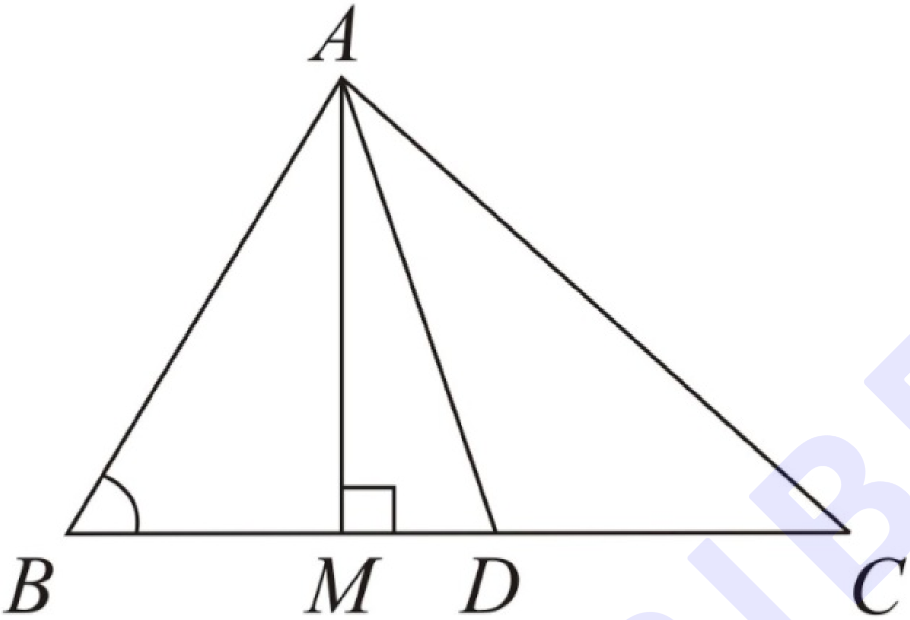
Solution:



In ΔABM, applying Pythagoras theorem
AB2=AM2+MB2

=AD2-DM2+MB2 =AD2-DM2+BD-MD2 =AD2-DM2+BD2+MD2-2BD.MD
=AD2+BD2-2BD.MD =AD2+BC22-2BC2×MD
AB2=AD2+BC22-BC×MD Hence, AB2=AD2-BC·DM+BC22

Q.9. In the figure, AD is the median of triangle ABC and AM⊥BC. Prove that:
AC2+AB2=2AD2+12BC2



Solution: In ΔAMB, by Pythagoras theorem,

AM2+MB2=AB2…(1)

In ΔAMC
AM2+MC2=AC2…(2)

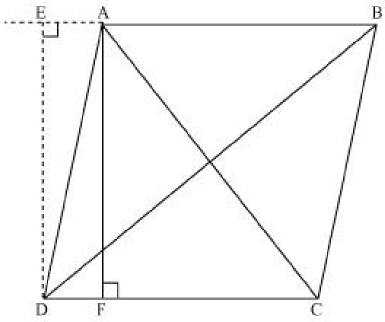
Adding equations (1) and (2)
2AM2+MB2+MC2=AB2+AC2

⇒2AM2+BD-DM2+MD+DC2=AB2+AC2

⇒2AM2+BD2+DM2-2BD.DM+MD2+DC2+2MD.DC=AB2+AC2
⇒2AM2+2MD2+BD2+DC2+2MD-BD+DC=AB2+AC2 ⇒2AM2+MD2+BC22+BC22+2MD-BC2+BC2=AB2+AC2
⇒2AD2+BC22=AB2+AC2 Hence, AC2+AB2=2AD2+12BC2

Q.10. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Solution:



Let ABCD be a parallelogram

Let us draw perpendicular DE on extended side BA and AF on side DC. In ΔDEA DE2+EA2=DA2…i In ΔDEB DE2+EB2=DB2 ⇒DE2+EA+AB2=DB2 ⇒DE2+EA2+AB2+2EA.AB=DB2 ⇒DA2+AB2+2EA.AB=DB2…(ii)

In ΔADF

AD2=AF2+FD2

In ΔAFC AC2=AF2+FC2 =AF2+DC-FD2 =AF2+DC2+FD2-2DC.FD =AF2+FD2+DC2-2DC.FD ⇒AC2=AD2+DC2-2DC.FD…(iii)

Since ABCD is a parallelogram

AB=CD and BC=AD

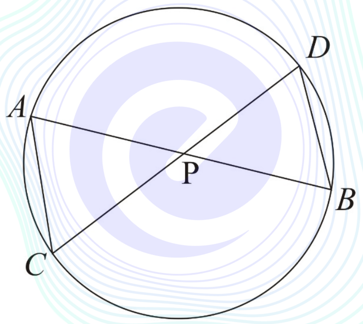
In ΔDEA and ΔADF ∠DEA=∠AFD ∠EAD=∠FDAEA∥DF ∠EDA=∠FADAF∥ED AD is common in both triangles. Since, respective angles are same and respective sides are same ΔDEA≅ΔAFD So, EA=DF

Adding equation (ii) and (iii)

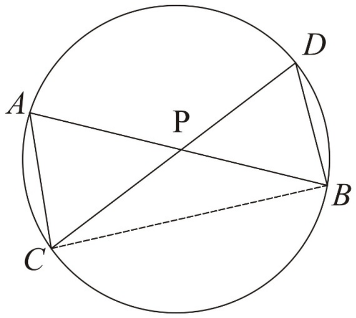
DA2+AB2+2EA.AB+AD2+DC2-2DC.FD=DB2+AC2

⇒DA2+AB2+AD2+DC2+2EA.AB-2DC.FD=DB2+AC2 ⇒BC2+AB2+AD2+DC2+2EA.AB-2AB.EA=DB2+AC2 ⇒AB2+BC2+CD2+DA2=AC2+BD2

Q.11. In Fig. 6.61, two chords AB and CD intersect each other at the point P. Prove that △APC~△DPB



Solution:

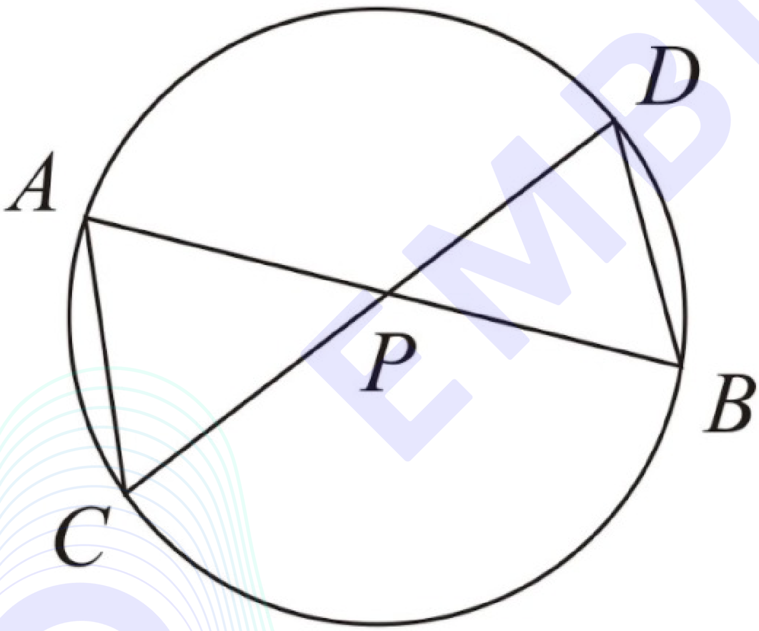


In $\triangle APC$ and $\triangle DPB$

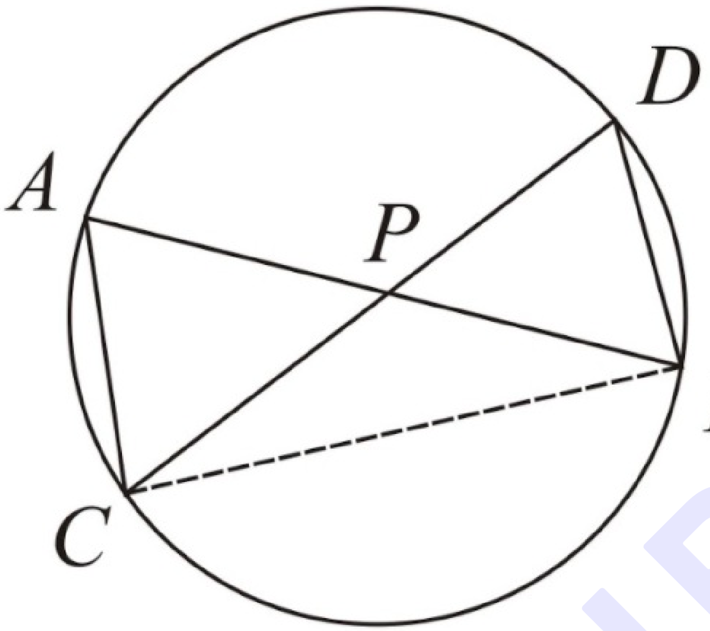
$\angle A = \angle D$ and $\angle C = \angle B$ (Angle on same segment)

Therefore, $\triangle APC \sim \triangle DPB$ (AA criteria)

Q.12. In figure two chords AB and CD intersect each other at the point P. Prove that $AP \cdot PB = CP \cdot DP$.



Solution:

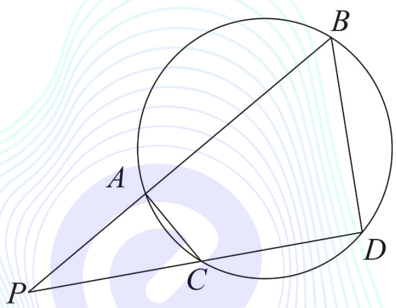


In $\triangle$APC and $\triangle$DPB

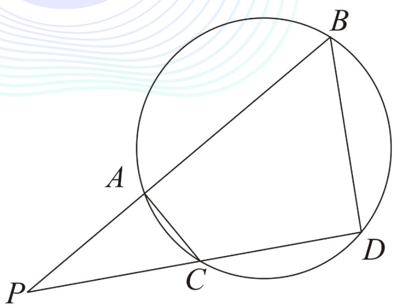
$\angle$A=$\angle$D and $\angle$C=$\angle$B (Angle on same segment)

Therefore, $\triangle$APC~$\triangle$DPB (AA criteria)
We know that corresponding sides of similar triangles are proportional
∴APDP=PCPB=CABD
⇒APDP=PCPB
∴ AP.PB=PC.DP

Q.13. In the figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that $\triangle$PAC~$\triangle$PDB.



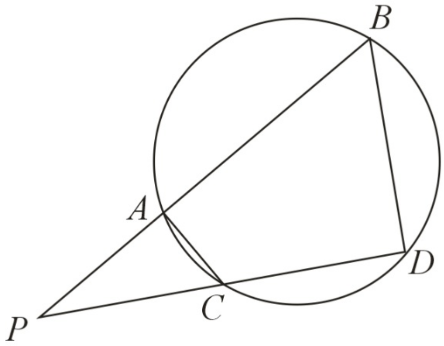
Solution:



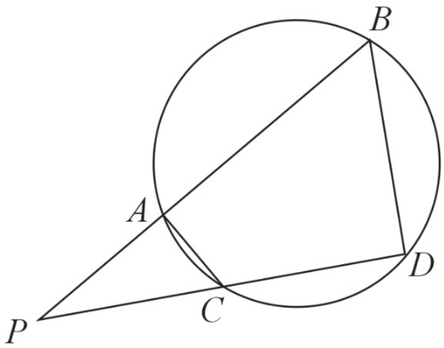
In $\triangle$PAC and $\triangle$PDB

$\angle$APC=$\angle$DPB (Common angle) $\angle$ACP=$\angle$DBP (Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.) Therefore, $\triangle$PAC~$\triangle$PDB (AA criteria)

Q.14. In the figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that PA.PB=PC.PD



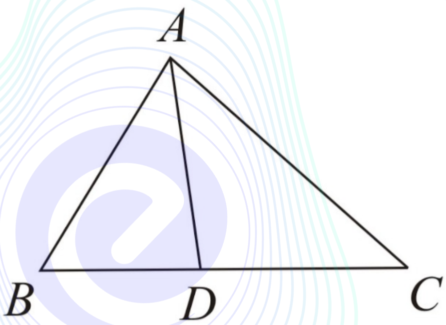
Solution:



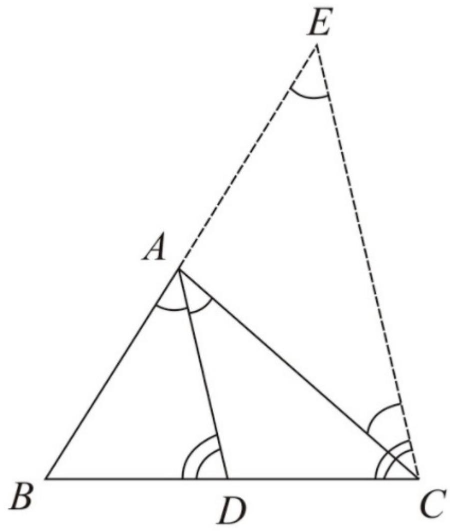
In △APC and △DPB

∠APC=∠DPB (Common angle) ∠ACP=∠DBP (Exterior angles of cyclic quadrilateral)
Therefore, △APC~△DPB (AA criteria) We know that corresponding sides of similar triangles are proportional.
PAPD=ACDB=PCPB
⇒PAPD=PCPB ∴PA.PB=PC.PD

Q.15. D is a point on side BC of ΔABC such that BDCD=ABAC. Prove that AD is the bisector of ∠BAC.



Solution:



Construct a line CE parallel to DA which meets BA produced at E.

Therefore, ∠BAD=∠BEC (Corresponding angles)......(1) ∠DAC=∠ACE (Alternate angles)......(2) In ΔDBA and ΔCBE, BDCD=ABAC (Given) ......(3) BDCD=BAAE (Basic proportionality theorem) ......(4) From (3) and (4), AE=AC Therefore, ∠ACE=∠BEC......(5) So, from (1), (2) and (5) ⇒∠BAD=∠DAC Therefore, AD is angle bisector of ∠BAC.